Radiant Energy Transfer Surface to Atmosphere

It can flow from the colder radiator to the warmer one

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Abstract

This article gives radiator properties for net radiant energy flow from the cooler to the warmer. \( \alpha_1 \varepsilon_0 < \alpha_0 \varepsilon_1 \). It explains why radiant energy flows two way. Flow from Earth’s surface absorbed by atmosphere exceeds that from atmosphere absorbed by surface. Introduction of more atmospheric CO\(_2\) increases atmosphere’s emissivity and absorptivity and probably cools atmosphere and surface slightly.

Introduction

The Global Warming from Green House Gas (CO\(_2\)) Theory, GHGT, controversy has been raging since 1990\(^{1,2}\). Proponents say radiant energy transfers from colder atmosphere with CO\(_2\) to Earths warmer surface, warming it (Fig. 1). Critics say since heat flows from higher temperature to lower, this must be wrong and the whole GHGT a hoax.

Many global warming experts claim radiant energy is transferred between two radiators at a rate proportional to their temperature difference (to power 4); much like thermal and convective heat transfer. There is confusion between electromagnetic energy intensity from a radiator (all matter emits) and energy transfer rate between two radiators (all matter absorbs, transmits and reflects).

This paper explains why this is only true in special, unrealistic cases. This allows the controversy to be resolved. More complete models have been published\(^3,4\).

Emission

Radiant energy, \( e_m \), is emitted from matter with intensity according to the Stefan-Boltzmann equation, \( I_i = \sigma \varepsilon_i T_i^4 \), where \( I_i \) = radiant exitance, radiant emittance, emission intensity, w/m\(^2\), rate per m\(^2\), a function of wavelength \( i \), \( T = \) surface temperature, deg K/100, \( \varepsilon_i = \) emissivity of radiator, a physical property of matter, function of wavelength \( i \). \( 0 < \varepsilon_i < 1 \), and \( \sigma \) = Stefan-Boltzmann fundamental constant of nature, 5.67 w/m\(^2\)//(K/100)\(^4\).

Many experts simplify with emissivity \( \varepsilon_i = 1 \) for both radiators, since emissivity is difficult to determine. This is the black body assumption. This is a source of controversy. Real matter has \( \varepsilon_i < 1 \).

By integrating over all wavelengths, \( i \), we can find a weighted average emissivity \( \varepsilon = \int \varepsilon_i \, di/i \) and corresponding average emission intensity \( I = \int I_i \, di/i = \int \sigma \varepsilon_i T_i^4 \, di/i = \sigma \varepsilon T^4 \). Therefore \( I, \, \text{w/m}^2 = 5.67 \varepsilon T^4 \) where \( \varepsilon \) is the average emissivity of the emitter and \( T \) is emitter temperature. This is the generalized Stefan-Boltzmann Equation for all matter radiators.
Absorption

Radiant energy is absorbed by matter with intensity that is the absorption fraction of incident radiation from the emitter. $J_i = \alpha_i I_0 = \alpha_i \sigma \varepsilon_0 T_0^4$, where

- $J_i$ = absorption intensity, w/m$^2$, rate per m$^2$, a function of wavelength $i$,
- $I_0$ = emitter intensity, w/m$^2$
- $T_0$ = emitter surface temperature, deg K/100,
- $\alpha_i$ = absorptivity of absorber, a physical property of matter, function of wavelength $i$. $0 < \alpha_i < 1$
- $\varepsilon_0$ = average emissivity of emitter
- $\sigma$ = Stefan-Boltzmann fundamental constant of nature, 5.67 w/m$^2$/(K/100)$^4$

The remaining incident radiation is transmitted through radiator, $\beta$, and reflected to surroundings, $\gamma$, such that $\alpha + \beta + \gamma = 1$.

Like emissivity, absorptivity has a spectrum of exitance vs. wavelength unique to each atom and molecule. By integrating over all wavelengths, $i$, we can find a weighted average absorptivity.
\( \alpha_1 = \int \alpha_i \, di/l \) and average absorption intensity \( J = \int J_i \, di/l = \int \sigma \, \alpha_i \, \varepsilon_0 \, T_0^4 \, di/l = \sigma \, \alpha_1 \, \varepsilon_0 \, T_0^4. \)

\( J_1 = \sigma \, \alpha_1 \, \varepsilon_0 \, T_0^4, \) where \( \alpha \) is average absorptivity and \( J \) is average absorption intensity. This is the fraction of incident em from emitter 0 absorbed by absorber 1.

**Energy Transfer Rate Law**

The driving force for net radiant energy transfer is an intensity difference between radiators, like a temperature difference for conduction and convection. In the latter two there is a temperature gradient through matter, transmitting the molecular/atomic kinetic energy (indicated by temperature). In the former case there is an electromagnetic energy field everywhere throughout space, transmitting electromagnetic energy among radiators in all directions by an intensity difference, gradient, among them.

The radiant energy transfer from radiator 0 to radiator 1 is intensity emitted by 0 and absorbed by 1. The energy transfer from radiator 1 to radiator 0 is the intensity emitted by 1 and absorbed by 0. This two-way energy transfer occurs simultaneously because radiation exists at many frequencies. The net energy transfer from 0 to 1 is the first less the second.

For simplicity we assume steady state from here. For unsteady state, just include energy accumulation rate, \( mCpdT/dt \), in the energy balance, input - output for the describing differential equation of the body of interest temperature. Latour\(^2\) gives a rigorous model of changing interacting energy flows to estimate radiator temperature changes.

For two insulated facing plate radiators 0 & 1: Transfer from 0 to 1 is fraction of incident em absorbed \( l_0 \, \alpha_1 \). Transfer from 1 to 0 is \( l_1 \, \alpha_0 \).

Emission intensities are \( l_0 = \sigma \, \varepsilon_0 \, T_0^4 \) and \( l_1 = \sigma \, \varepsilon_1 \, T_1^4 \). And absorption rates are \( J_0 = \sigma \, \alpha_0 \, \varepsilon_1 \, T_1^4 \) and \( J_1 = \sigma \, \alpha_1 \, \varepsilon_0 \, T_0^4 \). So, the net transfer is \( Q = l_0 \, \alpha_1 - l_1 \, \alpha_0 \).

The convention is, if \( Q > 0 \), energy transfers from 0 to 1; if \( Q < 0 \), energy transfers from 1 to 0. Of course, when \( Q = 0 \) there is no net transfer; two ways are equal. If \( Q = \) constant, the system is at steady state.

Substituting, \( Q_{0,1} = \sigma \, \alpha_1 \, \varepsilon_0 \, T_0^4 - \sigma \, \alpha_0 \, \varepsilon_1 \, T_1^4 = \sigma \, [\alpha_1 \, \varepsilon_0 \, T_0^4 - \alpha_0 \, \varepsilon_1 \, T_1^4]. \)

Only when \( \alpha_1 \, \varepsilon_0 = \alpha_0 \, \varepsilon_1 \) may we factor out \( \alpha_1 \, \varepsilon_0 \) to get \( Q_{0,1} = \sigma \, \alpha_1 \, \varepsilon_0 \, [T_0^4 - T_1^4]. \)

The general driving force\(^2\) for energy transfer between radiators 0 to 1 is

\( Q_{0,1} = \alpha_1 \, \varepsilon_0 \, T_0^4 - \alpha_0 \, \varepsilon_1 \, T_1^4; \) not \( T_0 - T_1. \)

This is the general equation for net radiant energy transfer between dissimilar radiators. The first term is the intensity from radiator 0 times the fraction absorbed by radiator 1, \( \alpha_1 \), i.e. flow
from 0 to 1. The second is the intensity from radiator 1 times the fraction absorbed by 0, \( \alpha_0 \), i.e. flow from 1 to 0.

Only when the two radiators’ emissivity and absorptivity products are equal may we say the driving force is a temperature difference, actually \([T_0^4 - T_1^4]\). This would happen when both radiators are of the same material.

Consider radiator 0 hotter than radiator 1: \( T_0 > T_1 \) or \( T_0 - T_1 > 0 \). Also assume radiator 0 has lower emissivity, \( \varepsilon_0 \), or higher absorptivity, \( \alpha_0 \), than 1 such that \( \alpha_1 \varepsilon_0 < \alpha_0 \varepsilon_1 \).

It follows from the general radiant energy transfer law \( Q_{0,1} = [\alpha_1 \varepsilon_0 T_0^4 - \alpha_0 \varepsilon_1 T_1^4] \) that \( Q_{0,1} < 0 \) when \( \alpha_0 \varepsilon_1 \) is sufficiently larger than \( T_0^4 \). In this case there is a solution \( T_1 < T_0 \). Energy transfers from colder \( T_1 \) to warmer \( T_0 \).

Example

Let \( T_0^4 = 6, T_1^4 = 4, T_0^4 > T_1^4 \)
Also let \( \alpha_1 \varepsilon_0 = 0.5, \alpha_0 \varepsilon_1 = 0.8 \).

\[
Q = [0.5 \times 6 - 0.8 \times 4] = (3 - 3.2) = -0.2 < 0.
\]

Here we have radiant energy flowing from the colder radiator \( T_1^4 = 4 \) to the hotter radiator \( T_0^4 = 6 \), because the colder has higher emissivity and lower absorptivity. (This does not prove GHGT Fig. 1 that net energy flows down from cold atmosphere to warmer surface. Just that it is theoretically possible with some physical property combinations.)

If both radiating surfaces obey Kirchhoff’s Law: \( \alpha_0 = \varepsilon_0 \) and \( \varepsilon_1 = \alpha_1 \), then \( \alpha_0 \varepsilon_1 = \alpha_1 \varepsilon_0 \).

Substituting \( Q_{10} = 5.67 [\alpha_0 \varepsilon_1 T_1^4 - \alpha_1 \varepsilon_0 T_0^4] = 5.67 [\varepsilon_0 \varepsilon_1 T_1^4 - \varepsilon_1 \varepsilon_0 T_0^4] = 5.67 \varepsilon_0 \varepsilon_1 [T_1^4 - T_0^4] \).

So, temperature difference driving force applies only to two Kirchhoff’s Law surfaces.

Simplifying further, assume both are black body radiators \( \alpha_0 = \varepsilon_1 = \alpha_1 = \varepsilon_0 = 1.0 \). We get a common expression \( Q_{10} = 5.67 [T_1^4 - T_0^4], > 0 \) because \( T_1 \) was selected \( > T_0 \), which is then used to claim radiant energy only transfers from the warmer radiator, \( T_1 \), to the cooler one, \( T_0 \).

Black bodies do not exist in nature; many real bodies do not obey Kirchhoff’s Law. These simplifications can be no better than an approximation, often a poor one.

Unsteady state transients

Energy balance of plate 0: Rate of accumulation in 0 equals input – output, where
\[
m_0 \ C_{p0} \ \frac{dT_0}{dt} = Q_0 - Q_{0,1}
\]
\( Q_0 = \) thermal energy input to 0
Q_{0,1} = \text{radiant energy transfer from 0 to 1 when } Q_{0,1} > 0. \text{ Otherwise rate is from 1 to 0.}

m_0 = \text{mass of plate, kg}

C_{p0} = \text{heat capacity of plate, joule/T \cdot g \cdot s}

t = \text{time, s.}

Rate of energy accumulation in 1 equals input – output, where \( m_1 \ C_{p1} \ \frac{dT_1}{dt} = Q_1 + Q_{0,1} \)

and where \( Q_1 = \text{thermal energy input to 1.} \)

This is a pair of coupled ordinary differential equations. With appropriate \( T_0 \) and \( T_1 \) initial conditions and specified input functions, \( Q_0(t) \) and \( Q_1(t) \), these can be solved for transient changes in \( T_0(t) \) and \( T_1(t) \).

These can be combined by addition, eliminating the em terms, \( m_0 \ C_{p0} \ \frac{dT_0}{dt} + m_1 \ C_{p1} \ \frac{dT_1}{dt} = Q_0 + Q_1 \). At steady state both derivatives are 0, and \( Q_0 = -Q_1 \).

Input rate = output rate = constant, naturally. One side is heated, and one side is refrigerated.

Each rate equals \( -Q_{0,1} \), which need not be 0. \( T_1 \) need not equal \( T_0 \) at steady state when each is constant.

**Application to Earth.**

We assume Earth’s atmosphere and surface are homogeneous plates with average properties radiating between each other at steady state.

Rate of energy transfer from surface 0 to atmosphere 1 is \( Q_{0,1} = \alpha_1 \ \varepsilon_0 \ T_0^4 - \alpha_0 \ \varepsilon_1 \ T_1^4 \)

To fit Fig 1, assume\(^2, ^4\)

\( T_0 = 2.88 \)

\( T_1 = 2.55 \)

\( \varepsilon_0 = 0.16407 \)

\( \varepsilon_1 = 0.083006 \)

\( \alpha_0 = 0.0875 \)

\( \alpha_1 = 0.40298 \)

\[
Q_{0,1} = 0.40298 \times 0.16407 \times 2.88^4 - 0.0875 \times 0.083006 \times 2.55^4
\]

\[
= 0.06612 \times 68.7971 - 0.007263 \times 42.2825
\]

\[
= 4.54865 - 0.30710 = \mathbf{4.24155} > 0
\]

Net radiant energy flows from 0 to 1; surface to atmosphere.

If radiating atmospheric CO\(_2\) increases, both \( \varepsilon_1 \) and \( \alpha_1 \) increase. To determine the change in these properties quantitively is beyond the scope of this paper.
Assume CO₂ increases from 400 to 800 ppmv and atmosphere properties increase 0.1% to:

ε₁ = 0.083006 * 1.001 = 0.083089
α₁ = 0.402980 * 1.001 = 0.40701
Q₀₁ = 0.06612 * 1.001 * 2.88⁴ - 0.007263 * 1.001 * 2.55⁴
   = 0.066186 * 68.7971 – 0.007270 * 42.2825
   = 4.55341 – 0.307406 = 4.24600 > 0 and > 4.24155

Both atmospheric emissivity and absorptivity increased with CO₂ doubling, but emissivity dominates. Net energy transfer rate from surface to atmosphere increased with CO₂, which would cool the surface. Connecting all interacting variables shows the same thing².

Conclusion

From the law of radiant energy transfer we have proven it can flow from the colder to warmer radiator, depending on emissivity and absorptivity properties of both radiators. And we have derived the specific physical property conditions of radiators where this can happen: the colder radiator has higher emissivity and lower absorptivity than the hotter radiator.

When T₀ > T₁ and α₁ ε₀ < α₀ ε₁, radiant energy flows from 1 to 0, provided Q₀₁ = α₁ ε₀ T₀⁴ - α₀ ε₁ T₁⁴ < 0

Using realistic average physical properties for Earth’s surface and atmosphere, we find energy transfers from surface to atmosphere at a higher rate than the reverse with increasing CO₂. This indicates net global cooling by radiant energy transfer from increasing atmospheric CO₂. This contradicts the Green House Gas Theory.

References